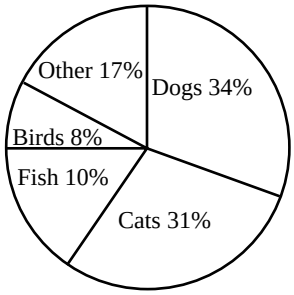
Junior High Sprint Round 10709

1. The International Cola Company uses 4 cups of sugar to make 50 cans of cola. If a case of cola has 200 cans, how many cups of sugar are needed to make 5 cases?

2. According to the graph, what percent of people have a favorite pet that isn't a dog or a cat?



Favorite Pets

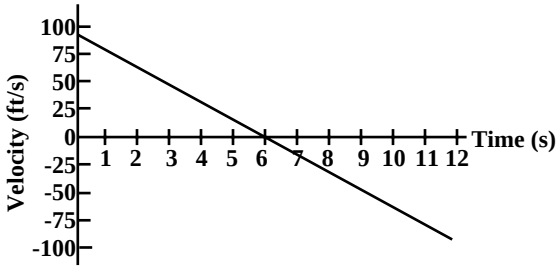
3. The Ariel Aquarium wants to buy some animals for a new exhibit. Dolphins need 75 m^3 of water each, walruses need 120 m^3, and whales need 345 m^3. If the aquarium wants four dolphins, one walrus and two whales, how many cubic meters of water will they need?

4. Three cats weigh the same as two dogs, and one dog weighs the same as six ferrets. The weight of how many ferrets is equal to that of one cat?

5. Ranchers in Texas earn $7.00 per hour. If a rancher works 5 hours per day, how many days will he have to work in order to earn $315?

6. A yard of concrete can be used to make a 5 foot by 7 foot porch. If you want to make a 20 foot by 28 foot porch, how many yards of concrete do you need?

7. John threw a ball up into the air. The graph below shows the vertical velocity of the ball after he threw it. After how many seconds did the ball start to come down again?



8. A museum charges $5 for adults and $2 for children. For every 10 children in a group, one adult gets in free. How much money will a group of 23 children and 4 adults need to pay to visit the museum?

9. Blair is twice as old as Wendy. Ten years ago he was three times as old as Wendy. How many years old is Blair now?

10. A cube of side length 3 is made from 27 unit cubes. The outside of the cube is then painted. How many of the unit cubes have exactly one face painted?

11. Let x be the sum of the positive factors of 12 and let y be the number of positive factors of 24. What is the value of $x - y$?

12. What is the maximum product of three distinct members of the set {-2, -1, 0.5, 1, 2}?

13. There are 20 first-graders at the 5th Street Elementary School. Twelve of the students like math and 7 of the students like reading. 5 students don't like math or reading. How many students like both?

14. Line p passes through the points (2,8) and (4,15). Line l is perpendicular to p. What is the slope of line l? Express your answer as a common fraction.

15. Families in five states were surveyed about how many cars they owned. 10,000 families in each state participated in the survey. The table below summarizes the responses. What percentage of the families surveyed own more than 2 cars?

	0 cars	1 car	2 cars	>2 cars
Iowa	750	2600	5850	800
California	300	2000	6200	1500
New York	1500	3500	4000	1000
Wisconsin	600	2500	6200	700
Nevada	800	4000	4700	500

16. Amanda earned an average of $60 per day on her first four days of work. After the fifth day her average was $62 per day. How much money did Amanda earn on the fifth day?

17. A sequence has the following rules: if a term in the sequence is prime, double it to get the next term. If it is not prime, add 3 to it instead. The first term in the sequence is 1. What percentage of the first 10 terms of the sequence are prime?

18. $a^2 + b^2 = 73$ and $ab = 24$. If both a and b are positive, what is the value of $a + b$?

19. If you flip 4 fair coins, what is the probability that exactly two of them will be heads? Express your answer as a common fraction.

20. How many numbers between 1000 and 2000 are multiples of 11?

21. Chris can run at 12 miles per hour, while Jayne only runs at 10 miles per hour. If it takes Jayne 72 minutes to run a race, how many minutes will it take Chris?

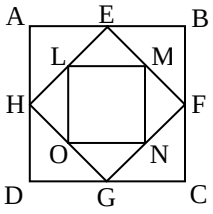
22. The measures of the interior angles of a pentagon form an arithmetic sequence. The smallest angle is $85°$. What is the degree measure of the second largest angle? Express your answer as a decimal to the nearest tenth.

23. How many positive integers less than 100 are multiples of 3 or 5 but not both?

24. Lily and Simon want to fill their pool. However, they only have buckets to carry the water with. Lily could fill the pool on her own in 6 hours, while it would take Simon 8 hours to fill the pool. How many hours would it take them to fill the pool if they worked together? Express your answer as a common fraction.

25. A basketball team has five forwards and seven guards. For a game, the team's starting lineup has two forwards and three guards. In how many different ways can the coach choose a starting lineup for the team?

26. Square ABCD has an area of 16. Square EFGH is inscribed in square ABCD such that each vertex of EFGH is a midpoint of one of the sides of ABCD. Square LMNO is inscribed in EFGH in a similar manner. What is the area of square LMNO?



27. $|2x| = |x - 2|$. What is the sum of all possible values of x? Express your answer as a decimal to the nearest tenth.

28. Randy is numbering the pages of his notebook. He only uses positive integers that are not prime or perfect squares. If he starts with the smallest possible number and goes in ascending order, what is the 20^{th} number that he writes?

29. 0.126126… is equivalent to what common fraction?

30. The planet Eris has 1¢, 2¢ and 5¢ coins. How many ways can someone make 15¢ using these coins?

Junior High Target Round 10709

1. The cube of a certain integer is 1a,bc4 and the fourth power of the same number is 3lm,n76, where the letters represent digits. What is the value of $a + b + c + l + m + n$?

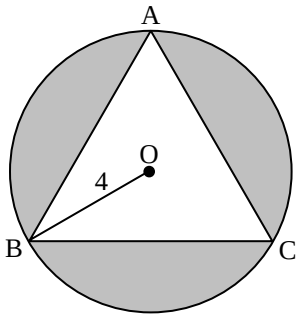
2. Farmer Ned needs to buy some new animals for his farm. Piglets cost $40 and calves cost $75. Farmer Ned buys a total of 50 animals, spending $2735. How many piglets does Farmer Ned buy?

3. The exchange rate between dollars and euros (€) is $1 = €0.871, while the exchange rate between dollars and yen (¥) is $1 = ¥117.3. If a new car costs 12,000 euros, how many yen would it cost? Round your answer to the nearest thousand yen.

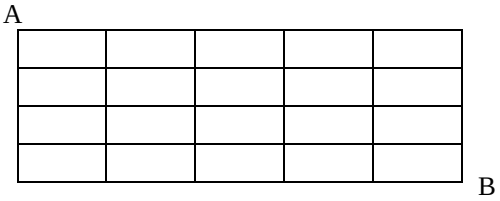
4. You roll two standard six-sided dice. What is the probability that the product of the two numbers rolled is greater than 15? Express your answer as a common fraction.

5. A palindrome is a number, like 1001, that reads the same if its digits are reversed. What is the positive difference between the number of five-digit palindromes and the number of four-digit palindromes?

6. Triangle ABC is an equilateral triangle inscribed in circle O of radius 4. What is the area of the shaded region in the diagram below? Express your answer as a decimal to the nearest tenth.



7. A rat is running along the grid shown below from point A to point B. The rat can move along any segment on the grid, but it can only go down or to the right. How many different ways are there for the rat to get from point A to point B?



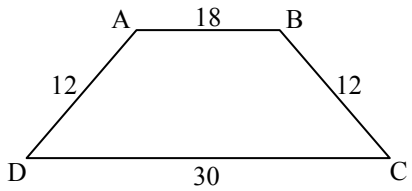
8. Louis and Becca are playing a game. Each person chooses an integer between 1 and 5, inclusive. They then take turns trying to guess the other person's number. The first person to guess correctly wins. Louis, like most people, guesses randomly each turn from among the remaining choices. Becca guesses each turn from among the remaining choices as well; however, she is twice as likely as someone guessing randomly to answer correctly. If Louis guesses first, what is the probability that Becca wins the game? Express your answer as a percentage.

Junior High Team Test Round 10709

1. If 1 meter is equal to 3.281 feet, how many meters are there in a mile? Express your answer as a decimal to the nearest tenth.

2. When selling a house, real estate agents earn a commission equal to 7% of the sale price of a property. If one agent works with the buyer and another with the seller, each of the agents earns 7%. If the two agents earn a combined commission of $55,440 and the house sold for 10% less than its assessed value, what was the assessed value of the house?

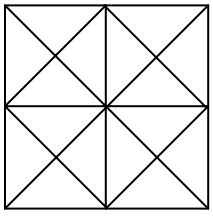
3. Equilateral trapezoid ABCD has bases of length 18 and 30. If the legs of the trapezoid have length 12, what is the trapezoid's area? Express your answer in simplest radical form.



4. A company wants to divide its employees into teams to work on a project. If they divide the employees into teams of four, they have three people left over. Similarly, if they divide the employees into teams of five they have four people left over, and if they divide them into teams of six they have five left over. If fewer than 100 people work for the company, how many employees does it have?

5. The prime factorization of 10! has the form $2^a 3^b 5^c 7^d$. What is the value of $a*b*c*d$?

6. How many squares are there in the drawing below?

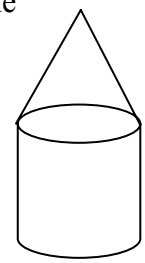


7. A sequence of numbers is generated with the following rule: starting with the fourth term, the n^{th} term, a_n, is given by the formula $a_n = a_{n-3} - a_{n-2} + a_{n-1}$. For example, if the first three terms were 2, 3, 4, then the fourth term would be 2 – 3 + 4 = 3. In this sequence, $a_1 = 4$ and $a_2 = -1$. What is the value of a_6?

8. Two spaceships are flying directly toward each other. One of the ships is flying at 50,000 km/hour, while the other ship is flying at 80,000 km/hour. When the ships are 400,000 km apart, one of the ships launches a probe. The probe travels at 120,000 km/h and flies continuously back and forth between the two ships. By the time the two ships meet, how far has the probe flown? Round your answer to the nearest thousand km.

9. Tracy's clock does not tell time accurately. When the hands on Tracy's clock show that an hour has passed, the real time has only changed by 55 minutes. Tracy's clock is currently showing the correct time. If Tracy has a 12-hour clock, how many hours will it take before her clock next shows the correct time?

10. The object on the right consists of a cone on top of a cylinder. The cone and the cylinder have equal heights and radii. The total height of the object is twice its radius. If the total volume of the object is 288π in^3, what is the diameter of the object in inches?



Junior High Countdown Round 10709

1. A factory can produce 120 bicycles in an hour. How many minutes would it take for the factory to produce 200 bicycles?

2. If $4x - 2y = 12$, what is the value of $-10x + 5y$?

3. Finnegan drove on the highway for six hours. If he never broke the speed limit of 75 mph, what is the largest distance, in miles, that he could have traveled?

4. If (x, y) is the midpoint of the line segment with endpoints (2, 8) and (-4, 12), what is the value of $x + y$?

5. What is the smallest four-digit number that is a multiple of 9?

6. I was born on a Saturday. What is the probability that my two best friends were also born during a weekend? Express your answer as a common fraction.

7. A rectangle's length is three times its width. If the area of the rectangle is 363 square feet, what is the length of the rectangle in feet?

8. If $a \Diamond b$ is defined as $a \Diamond b = b^2 - 2ab$, what is the value of $(-1) \Diamond (2 \Diamond 3)$?

9. What is the value of 29 * 31?

10. If 60% of the ninth graders in Sioux City are girls and there are a total of 2500 ninth grade students, how many boys are there in the ninth grade?

11. What is the sum of the first 50 positive integers?

12. What is the smallest positive integer that has exactly five factors?

13. A circular dartboard has a radius of 8 inches. The bulls-eye at the center has a radius of 2 inches. If a dart hits a random place on the dartboard, what is the probability that it doesn't land in the bulls-eye? Express your answer as a common fraction.

14. If $x = 2$ and $y = 3$, what is the value of x^{-3} / y^{-2} in simplest terms?

15. Ten years from now Sam will be twice as old as she was three years ago. How many years old is Sam now?

16. What is the slope of the line $6x - 2y = 13$?

17. Jason's desk drawer has six red, eight blue and twelve black pens. If Jason randomly removes pens from the drawer, how many does he need to take in order to guarantee that he gets at least four pens of the same color?

18. Two sides of a triangle have lengths 5 and 8. How many different integer lengths can the third side of the triangle have?

19. A graph has the equation $y = x^2 - 19$. If the *y*-coordinate of a point is 125, what is the sum of the possible *x*-coordinates of the point?

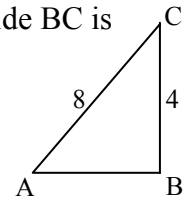
20. How many integer values of *x* satisfy the inequality $-4 < x + 12 \leq 6$?

21. The median of three numbers is 3, and their mean is 5. What is the minimum value of the largest number?

22. Leah saved $300 in January. Every month she saves $35 more than she did during previous month. How much money did Leah save in July?

23. If the chance of rain on Monday is 0.4 and the chance of rain on Tuesday is 0.2, what is the probability that it won't rain on either of the days? Express your answer as a decimal to the nearest hundredth.

24. Right triangle ABC has a hypotenuse of length 8. If the length of side BC is 4, what is the measure of angle C in degrees?



25. If there are 8 furlongs in a mile and 40 rods in a furlong, a stretch of road measuring 22 furlongs and 240 rods is how many miles long? Express your answer as a decimal to the nearest tenth.

26. Express $\sqrt{98} * \sqrt{27}$ in simplest radical form.

27. Two hamburgers and an order of fries cost $3.85. If a hamburger costs $1.25, how much does an order of fries cost?

28. If $27^2 + 9^3 + 3^6 = 3^x$, what is the value of *x*?

29. Adam and Zero both have favorite numbers that are prime. If Zero's favorite number is three greater than Adam's, what is the product of their favorite numbers?

30. What is the arithmetic mean of 25, 14 and 60?

31. What is the maximum number of regions that the interior of a circle can be divided into by three lines?

32. What is the value of $(1+x)(2+x)\cdots(20+x)$ if $x=-8$?

33. Number stickers come in strips containing one of each of the digits zero through nine. How many strips are needed to write all of the numbers from 1 to 50 inclusive?

34. If $\langle x, y\rangle$ is defined as $\langle x, y\rangle = 2xy - 4$, what is the value of $\langle 3,4\rangle - \langle -2,1\rangle$?

35. The tax on a gallon of gasoline is 48¢. If the price of a gallon of gasoline, including tax, is \$3.20, what percent of the price is tax?

36. It costs \$38 to ride a train for up to 80 miles, and \$2.50 for every additional ten miles. How many dollars does a 160 mile train ride cost?

37. A rectangular garden is eight feet long and six feet wide. There is a paved walkway around the edge of the garden that is two feet wide. What is the area of the walkway in square feet?

38. What is the sum of the positive, single-digit factors of 56?

39. What is the surface area, in square feet, of a cube with a volume of 64 cubic feet?

40. How many two-digit numbers have units digits that are four greater than their tens digits?

41. If $f(x) = (x^2 - 3x + 1)^{-1}$, what is the value of $f(4)$? Express your answer as a common fraction.

42. There are 12 people at a party, and each person shakes hands with everyone else exactly once. What is the total number of handshakes that occur at the party?

43. What is the area of a square inscribed in a circle of radius 3?

44. William borrows money from the bank in order to buy a new computer. He pays the bank \$180 per month for 40 months to pay off the loan. If the computer cost \$5400, what percentage of the loan was interest?

45. How many games are played in a single-elimination tournament with 32 competitors?

46. What is the surface area, in square units, of a cylinder with radius 2 units and height 3 units? Express your answer in terms of π.

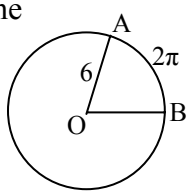
47. Alice gets an allowance of \$12 every Sunday. If the first day of July is a Sunday, how many dollars will Alice get in July and August?

48. How many zeros are there at the end of the product of 8^2 and 25^4?

49. If $a \otimes b$ is defined as $a \otimes b = 2a - 3b$, what is the value of $(6 \otimes 1) \otimes (-3 \otimes -4)$?

50. How many unique sums can be made by adding two numbers from the set $\{2,3,4,5,6\}$?

51. Circle O has a radius of 6 units. If the length of arc AB is 2π units, what is the measure of angle AOB in degrees?



52. There are 90 marbles in a pile. If Steve takes 1/2 of the pile and Iain takes 1/3 of the remaining marbles, how many marbles are left in the pile?

53. Torin has six days to read a book. He reads one page on the first day, two pages on the second day, and he continues to double the number of pages that he reads each day. If Torin reads exactly enough to finish the book in six days, how many pages does the book have?

54. If $a + b + c = 15$, $a - b = 4$ and $a - c = 20$, what is the value of b?

55. How many ways are there to choose a president and two vice presidents from a group of six people?

56. What fraction of the positive odd numbers less than 30 are prime? Express your answer in simplest terms.

57. If 30% of 120% of a number is 18, what is the number?

58. A cone has a radius of 5 inches and a height of 4 inches. If a cylinder has the same radius but twice the height, the volume of the cylinder is how many times the volume of the cone?

59. When the product $(2x - 4)(x + 3)$ is expanded, what is the sum of the coefficients of x and x^2?

60. Chuck drove the 60 miles from his house to the airport at an average speed of 30 mph. He averaged 20 mph when he returned along the same route. What was Chuck's average speed in mph for the entire trip?

61. Travis has $3.60 in quarters and nickels in his piggy bank. If Travis has seven quarters, how many nickels does he have?

62. What is the area of the triangle formed by the line $x = 5$, the line $y = x$ and the x-axis? Express your answer as a decimal to the nearest tenth.

63. If the sum of an integer n and its cube is –68, what is n?

64. What is the coefficient of x^2y^2 in the expansion of $(x + y)^4$?

65. How many different four-digit numbers can be made by using each of the digits 0, 3, 5 and 8 exactly once?

66. A store sells clothing for 40% more than the wholesale price. If the store sold a pair of jeans for $84, what was the wholesale price in dollars?

67. A rhombus has diagonals of length 12 inches and 16 inches. What is the area of the rhombus in square inches?

68. How many positive factors does 128 have?

69. A bag of 8 potatoes costs $2.24, while loose potatoes are sold for 34¢ each. How much cheaper is a bagged potato than a loose one, in cents per potato?

70. How many integer values of x satisfy the equation $|2x| < 8$?

71. What is the probability of drawing two clubs, without replacement, from a standard 52-card deck? Express your answer as a common fraction.

72. A sphere has a surface area of 36π cm^2. What is the radius of the sphere in cm?

73. What is the smallest number greater than one that is both a perfect cube and a perfect square?

74. It takes four weeks to build the first floor of an apartment building, and each additional floor takes two weeks longer to build than the one below it. How many weeks will it take to build an apartment building with five floors?

75. How many different solution sets (x, y) are there to the equation $x + y = 13$ if both x and y are non-negative integers?

76. How many different ways are there to arrange 5 people in a straight line?

77. Iain's car gets 24 miles per gallon while Emily's car gets 30 miles per gallon. If they both drive for 1200 miles, how many more gallons of gas will Iain's car need than Emily's?

78. What is the area of a right triangle with hypotenuse of length 25 and one leg of length 7?

79. What is the greatest common factor of 28, 70 and 84?

80. If $3^{3x}4^{y} = 6^{6}$, what is the value of $x + y$?

Junior High Round 10709 Solutions

Sprint Round

1. 80
2. 35 (%)
3. 1110
4. 4
5. 9
6. 16
7. 6
8. ($)56
9. 40
10. 6
11. 20
12. 4
13. 4
14. -2/7
15. 9 (%)
16. ($)70
17. 30 (%)
18. 11
19. 3/8
20. 91
21. 60
22. 119.5
23. 40
24. 24/7
25. 350
26. 4
27. -1.3
28. 38
29. 14/111
30. 18

Target Round

1. 24
2. 29
3. (¥) 1,616,000
4. 11/36
5. 810
6. 29.5
7. 126
8. 60 (%)

Team Round

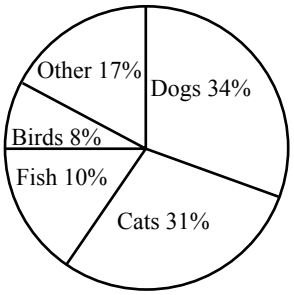
1. 1609.3
2. ($)440,000
3. $144\sqrt{3}$
4. 59
5. 64
6. 10
7. -1
8. 369,000
9. 132
10. 12

Countdown Round

1. 100
2. -30
3. 450
4. 9
5. 1008
6. 4/49
7. 33
8. 3
9. 899
10. 1000
11. 1275
12. 16
13. 15/16
14. 9/8
15. 16
16. 3
17. 10
18. 9
19. 0
20. 10
21. 9
22. ($)510
23. 0.48
24. 60(°)
25. 3.5
26. $21\sqrt{6}$
27. $1.35
28. 7
29. 10
30. 33
31. 7
32. 0
33. 15
34. 28
35. 15(%)
36. ($)58
37. 72
38. 22
39. 96
40. 5
41. 1/5
42. 66
43. 18
44. 25(%)
45. 31
46. 20π
47. ($)108
48. 6
49. 0
50. 7
51. 60(°)
52. 30
53. 63
54. 9
55. 60
56. 3/5
57. 50
58. 6
59. 4
60. 24
61. 37
62. 12.5
63. -4
64. 6
65. 18
66. ($)60
67. 96
68. 8
69. 6(¢)
70. 7
71. 1/17
72. 3
73. 64
74. 40
75. 14
76. 120
77. 10
78. 84
79. 14
80. 5

Sprint Round

1. There are 5 cases and 200 cans per case, so there are a total of 5 x 200 = 1000 cans. 50 cans need 4 cups of sugar. There are 1000/50 = 20 groups of 50 cans, so 4 * 20 = 80 cups of sugar are needed. **Answer: 80 cups**

2. 

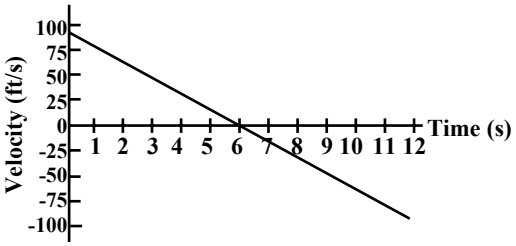
Favorite Pets
The three categories that aren't dogs or cats are other, birds and fish. The sum of their percentages is 17% + 8% + 10% = 35% **Answer: 35%**

3. The total volume required is the sum of the volumes for the individual animals. Thus, 4 * 75 + 1 * 120 + 2 * 345 = 1110 m^3 of water is required. **Answer: 1110 m^3**

4. We can call the weight of a cat c, that of a dog d, and that of a ferret f. We then have the equations $3c = 2d$ and $d = 6f$. Multiply the right-hand equation by 2 to get $2d = 12f$. Combining this with the left-hand equation, $3c = 12f \rightarrow c = 4f$. So, one cat weighs the same as 4 ferrets. **Answer: 4 ferrets**

5. Ranchers earn \$7 * 5 = \$35/day. In order to earn \$315, they must work 315/35 = 9 days. **Answer: 9 days**

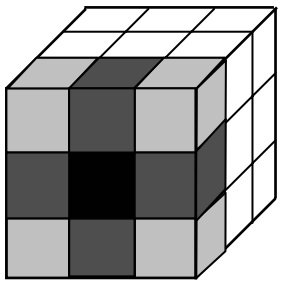
6. 1 yard of concrete can make a 3 * 5 = 35 ft^2 porch. The area that we want is 20 * 28 = 560 ft^2. Thus, we need 560/35 = 16 yards of concrete. It is easier if you notice that the two shapes are similar. The sides of the larger porch are 4 times as long as the sides of the smaller, so the area is $4^2 = 16$ times larger. **Answer: 16 yards**

7. 

When the ball is moving up, it has a positive velocity. On the way down, it has a negative velocity. Therefore, the point where the velocity goes from positive to negative (v = 0) occurs when the ball starts to come down. From the graph, this occurs at the 6-second mark. **Answer: 6 seconds**

8. 23/10 = 2.3, so there are 2 full groups of 10 children. This means that 2 adults get in free. This leaves 23 children and 2 adults that have to pay. The cost is \$5 * 2 + \$2 * 23 = \$56. **Answer: \$56**

9. Let Blair's age be b and Wendy's age be w. The first sentence gives the equation $b = 2w$. Ten years ago their ages were $b - 10$ and $w - 10$, respectively. The second sentence then gives the equation $b - 10 = 3(w - 10)$. Substituting in the first equation gives $2w - 10 = 3(w - 10)$. Then: $2w - 10 = 3w - 30 \rightarrow 20 = w$. Since we need Blair's age, we use the first equation again. $b = 2w = 2(20) = 40$. **Answer: 40 years**

10. 
We will look at just the front face of the cube to start with. There are 3 * 3 = 9 unit cubes with faces exposed here. As we can see from the drawing, the 4 corner cubes (like the upper-right one) have 3 faces painted, shown in light gray. There are 4 other cubes, shown in dark gray, that have 2 faces painted. There is just one unit cube, the black one, which has one face painted. This is true on each of the other faces as well. Since there are 6 faces, there are a total of 6 * 1 = 6 unit cubes with one face painted. **Answer: 6 cubes**

11. We begin by listing the factors of 12: 1*12, 2*6, 3*4 all make 12, so 12 has six factors. Their sum

is 1 + 12 + 2 + 6 + 3 + 4 = 28. Thus, $x = 28$. We now need to list the factors of 24. 1*24, 2*12, 3*8 and 4*6 all make 24. Thus, 24 has 8 factors and $y = 8$. From these two values, we know that $x - y = 28 - 8 = 20$. **Answer: 20**

12. The given set is {-2, -1, 0.5, 1, 2}. To have a positive product, we need to either multiply 3 positive numbers or 1 positive number and 2 negative ones (remember that the product of 2 negative numbers is positive). The only product of 3 positive numbers is 0.5 * 1 * 2 = 1. The product of the two negative numbers is -2 * -1 = 2. To get the biggest product, we should multiply this by 2, the largest positive number. This gives -2 * -1 * 2 = 4. Multiplying by 1 or 0.5 would have given a smaller product. The maximum product is thus 4. **Answer: 4**

13. 5 of the students don't like math or reading, leaving 20 – 5 = 15 students that like at least one of the subjects. 12 of these students like math, leaving 15 – 12 = 3 students that don't like math. Since all of the 15 students like math or reading, these 3 students must like reading. A total of 7 students like reading, so after these 3 there are still 7 – 3 = 4 other students that like reading. These 4 students must be in the 12 students that like math (they can't be in the 5 that don't like either subject). So, 4 students must like math and reading. **Answer: 4 students**

14. The equation for the slope of a line is change in y divided by change in x. With two points (x_1, y_1) and (x_2, y_2) this is given by the equation slope = $(y_2 - y_1)/(x_2 - x_1)$.
Plugging our two points (2, 8) and (4, 15) into the equation gives:
slope = (15 – 8)/(4 – 2) = 7/2.
We now need to find the slope of l, which is perpendicular to this line. The rule for perpendicular lines is that the product of their slopes is -1. So, if we say the slope of l is m, then 7/2 * m = -1 or m = -2/7. **Answer: -2/7**

15.

	0 cars	1 car	2 cars	>2 cars
Iowa	750	2600	5850	800
California	300	2000	6200	1500
New York	1500	3500	4000	1000
Wisconsin	600	2500	6200	700
Nevada	800	4000	4700	500

We are interested in how many families own more than 2 cars, the far right column. Adding the values in this column gives
800 + 1500 + 1000 + 700 + 500 = 4,500 families.
With 10,000 families in each of 5 states, there were a total of 10,000 * 5 = 50,000 families surveyed. We want to know what percentage of this value 4,500 families is. This is given by 4500/50000 * 100 = 9/100 * 100 = 9%. **Answer: 9%**

16. Amanda earned an average of $60 each day for the first four days. This means that she earned a total of $60/day * 4 days = $240 during this time. Likewise, her average of $62/day during the 5 days means that she earned $62 * 5 = $310 during the entire 5 days. Her earnings on the fifth day are $310 – $240 = $70. **Answer: $70**

17. The main thing here is just to check if the numbers in the sequence are prime, and then follow the appropriate rule. The first number is 1, and 1 is **not** a prime number, so the next term is 1 + 3 = 4. This is also not prime, so the third term is 4 + 3 = 7. Seven is prime, so the fourth term is 7 * 2 = 14. Following these rules gives the first 10 terms of the sequence as shown below, with prime terms underlined:
1, 4, **7**, 14, **17**, 34, **37**, 74, 77, 80.
We want to know what percent of the terms are prime. There are 3 prime terms out of 10 total, so 3/10 * 100 = 30% of the terms are prime.
Answer: 30%

18. For this problem, we use the fact that $(a+b)^2 = a^2 + 2ab + b^2$. We know that $a^2 + b^2 = 73$ and $ab = 24$. Inserting these values into the first equation gives $(a+b)^2 = 73 + 2*24 = 121$. Taking the square root of this gives $a + b = \pm 11$. Since both a and b are positive, $a + b$ must equal 11. **Answer: 11**

19. Flipping heads or tails is equally likely, so each flip has a 1/2 chance of landing in a given direction. So, the probability of flipping a specific sequence of heads and tails like HTTH is (1/2) * (1/2) * (1/2) * (1/2) = $(1/2)^4$ = 1/16. We now need to know how many sequences of 4 flips there are with exactly two heads. We can list them rather easily: HHTT, HTHT, HTTH, THHT, THTH, TTHH. There are six ways of flipping two heads, and each has a probability of 1/16, so the overall probability is 6 * 1/16 = 6/16 = 3/8. **Answer: 3/8**

20. The smallest multiple of 11 that is larger than 1000 is 11 * 91 = 1001. The largest multiple of 11 that is less than 2000 is 11 * 181 = 1991.

There are 181 – 91 + 1 = 91 numbers in the range from 91 to 181 (it is important to remember to add in the extra 1 after subtracting). Thus, there are 91 multiples of 11 between 1000 and 2000. **Answer: 91**

21. If Jayne runs at 10 miles per hour, then it takes him 60/10 = 6 minutes to run one mile. Since it took him 72 minutes to run the whole race, then the race is 72/6 = 12 miles long. Since Chris runs at 12 mph, it will take him exactly one hour, or 60 minutes, to run the race. **Answer: 60 minutes**

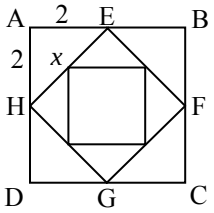
22. The sum of the measures of the interior angles of a polygon with n sides is 180° * (n – 2). For a pentagon n = 5, so the value is 180° * (5 – 2) = 180° * 3 = 540°. The average measure of an angle is then 540°/5 = 108°. Because the angles form an arithmetic sequence with 5 terms, the measure of the middle angle is equal to the average of 108°. The measure of the smallest angle is 85°. These values are separated by twice the common difference of the sequence, so this common difference is (108° – 85°)/2 = (23°)/2 = 11.5°. We want the measure second largest angle, which is the fourth in the sequence, one after the middle term. So, the measure of this angle is 108° + 11.5° = 119.5°. **Answer: 119.5°**

23. The largest multiple of 3 that is less than 100 is 33 * 3 = 99, so there are 33 multiples of 3 less than 100. Likewise, 19 * 5 = 95, so there are 19 multiples of 5 that are less than 100. However, both of these values include numbers that are multiples of both 3 and 5. We need to find these and remove them. 3 and 5 have no common factors (they are *relatively prime*) and so their least common multiple is 3 * 5 = 15. Since 6 * 15 = 90, there are 6 multiples of 15 less than 100. We counted these in both the multiples of 3 and the multiples of 5, so we need to subtract these 6 multiples from each of our totals. This gives 33 – 6 = 27 multiples of 3 and 19 – 6 = 13 multiples of 5 that are not multiples of 15. Adding these values gives 27 + 13 = 40 numbers that fit our criteria. **Answer: 40**

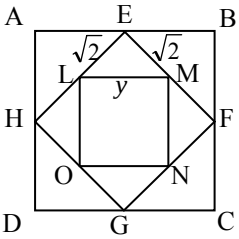
24. If Lily can fill the pool in 6 hours, then in one hour she can fill 1/6 of the pool. Likewise, Simon can fill 1/8 of the pool in an hour. Working together, they can fill 1/6 + 1/8 = 7/24 of the pool in an hour. If they fill the entire pool in h hours, then we have the equation (7/24) * h = 1 → h = 24/7. **Answer: 24/7 hours**

25. Let's look at the forwards first. There are 5 ways to choose the first forward and 4 ways to choose the second one, giving a total of 5 * 4 = 20 ways. However, it doesn't matter what order the forwards are chosen in, choosing player A and then player B is the same as choosing B and then A. So, we actually have half as many ways to choose the forwards, only 10.
We now do the same thing for the guards. There are 7 * 6 * 5 = 210 ways to choose three of the seven players. However, again we are counting lineups several times. There are six ways to choose the same three players: ABC, ACB, BAC, BCA, CBA. So, there are actually 210/6 = 35 ways to choose the guards. To get the total number of lineups, we multiply the number of ways to choose the forwards by the number of ways to choose the guards, getting 35 * 10 = 350 lineups.
Using combinations there is the simple equation: $\binom{5}{2} * \binom{7}{3} = \frac{5!}{2!3!} * \frac{7!}{3!4!} = 10 * 35 = 350$.
Answer: 350

26. Since the area of ABCD is 16, the length of a side is $\sqrt{16} = 4$. Since EFGH is also a square, E is the midpoint of AB and H is the midpoint of AD. This means that AE = AH = 2. If we let the length of EH be x, we have the following situation:

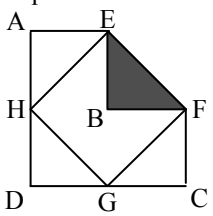


From the Pythagorean theorem, we have $x^2 = 2^2 + 2^2 = 8$ → $x = \sqrt{8} = 2\sqrt{2}$. We now repeat the process to find the length of LM. We have EL = EM = $2\sqrt{2}/2 = \sqrt{2}$. Letting LM = y:



Again, from the Pythagorean theorem we get $y^2 = \left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2 = 2 + 2 = 4$. Since the area of LMNO is also equal to y^2, our answer is 4.

There is another solution to the problem that doesn't require the Pythagorean theorem. Looking at just the outer two squares, you can see that one corner of the larger square can fold in to cover a quarter of the smaller one.



If this is done with all four corners then they will entirely cover the inner square. This means that square ABCD covers square EFGH exactly twice, so the area of EFGH is half the area of ABCD. The relationship between LMNO and EFGH must be the same. So, the area of LMNO is 1/2 * 1/2 = 1/4 the area of ABCD. This area is 1/4 * 16 = 4. **Answer: 4**

27. If both of the quantities within the absolute values are positive, then we have $2x = x - 2 \rightarrow x = -2$.
If we assume that the quantity on the left is negative, then
$-2x = x - 2 \rightarrow -3x = -2 \rightarrow x = 2/3$.
These are the only two possible values of x.
Assuming that the quantity on the right is negative and the left positive is the same as multiplying the second equation by -1, while assuming that both quantities are negative is the same as multiplying the top equation by -1. Neither of these changes the solutions. So, the sum of the solutions is -2 + 2/3 = -1.333...
Rounding this to the nearest tenth gives -1.3
Answer: -1.3

28. The easiest way to do this is to just list the first 40 or so positive integers and cross off the ones that are either prime or perfect squares.
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40
Now, it is just a matter of counting the numbers that haven't been removed (in black) and finding the 20th one. **Answer: 38**

29. Let $x = 0.126126\ldots$ Multiplying both sides by 1000 gives $1000x = 126.126126\ldots$ Notice that these numbers are identical after the decimal point. Subtracting one from the other gives

$$\begin{aligned} 1000x &= 126.126126\ldots \\ -\quad x &= \quad 0.126126\ldots \\ \hline 999x &= 126 \end{aligned}$$

Solving this for x gives $x = 126/999$. However, 126 and 999 share a common factor of 9, so to make it a common fraction we need $x = 14/111$.
Answer: 14/111

30. We will count the number of ways by first splitting them into groups based on how many 5¢ coins are used. Within each group, we can then see how many different amounts of 2¢ coins can be added while keeping the total less than or equal to 15¢, using 1¢ coins to fill in the rest.
With three 5¢ coins we already have 15¢, so there is only one way to do this.
With two 5¢ coins we have 10¢. We can add two 2¢ coins to bring the total to 14¢. We can also use one or zero 2¢ coins, with totals of 12¢ and 10¢ respectively. (Remember that we use the 1¢ coins to bring the totals up to 15¢.) So there are three ways to make 15¢ in this group.
With one 5¢ coin we have 5¢. Adding five 2¢ coins brings the total to 15¢. We can also use any number of 2¢ coins less than this. So we can have 0 to 5 of the 2¢ coins, giving us 6 ways to make 15¢.
For the final group we have no 5¢ coins. We can use at most seven 2¢ coins, giving us a total of 14¢. So, we have between 0 and 7 of the 2¢ coins, giving us 8 ways to make 15¢.
Adding the number of ways from these four groups gives 1 + 3 + 6 + 8 = 18. **Answer: 18 ways**

Target Round

1. Let our integer be x. From the problem, we know that x^3 is between 10,000 and 20,000. Taking the cube roots of these numbers, we find that x is between 21.5 and 27.1. We also know that x^3 ends in a 4. The only integers with cubes that end in 4 have a 4 in their units digits. Combining these two pieces of information, we get $x = 24$. $24^3 = 13{,}824$ and $24^4 = 331{,}776$. We can now add the missing digits from the problem, getting 3 + 8 + 2 + 3 + 1 + 7 = 24. **Answer: 24**

2. Let p be the number of piglets that Ned buys, and let c be the number of calves. He buys 50 animals total, so we have $p + c = 50$. We also know that piglets cost $40 and calves cost $75, and Ned spends a total of $2735. This gives us the equation $40p + 75c = 2735$. Substituting: $c = 50 - p$

$40p + 75(50 - p) = 2735$
$40p + 3750 - 75p = 2735$
$-35p = -1015$
$p = 29$
Answer: 29 piglets

3. We first need to convert from euros to dollars. \$1 = €0.871, so €1 = \$1/0.871 ≈ \$1.148. We can now convert our price to dollars using this exchange rate: €12,000 * \$1.148/€ = \$13,776. From here, we need to convert the dollars to yen. For this we have the exchange rate \$1 = ¥117.3. We can use this exchange rate right away, getting \$13,776 * ¥117.3/\$ = ¥1,615,924.8, which rounds to ¥1,616,000 as asked for in the problem. **Answer**: **¥1,616,000**

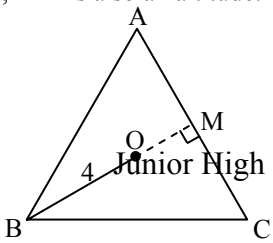
4. We can easily make a table showing all of the possible products from our two dice.

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

The numbers below the dark line are all of the products greater than 15. There are 11 such products. Since there are a total of 36 products, the probability that a randomly rolled product is greater than 15 is 11/36. **Answer: 11/36**

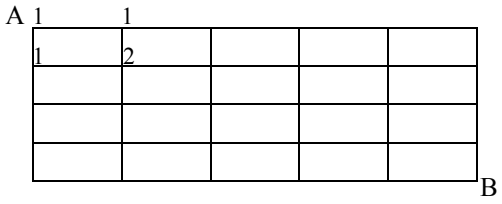
5. All four-digit palindromes have the form *abba* (this comes from the definition of a palindrome). *a* can be any digit except 0, because 0 would make *abba* only a three-digit number. For each *a*, *b* can be any of the digits 0-9. Thus, there are 9 choices for *a* and 10 for *b*, giving a total of 9*10 = 90 four-digit palindromes. Likewise, five-digit palindromes have the form *abcba*. Again, there are 9 choices for *a*, and 10 choices for each of *b* and *c*. Therefore, there are 9*10*10 = 900 five-digit palindromes. This means that there are 900 – 90 = 810 more five-digit palindromes than four-digit ones. **Answer: 810**

6. The first task in this problem is finding the area of triangle ABC. We extend the segment $\overline{BO}$ until it intersects $\overline{AC}$ at a new point M. $\overline{BM}$ is a median of ABC. Since the triangle is equilateral, $\overline{BM}$ is also an altitude.

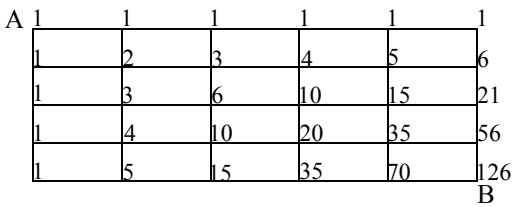


There is a theorem that says that in this situation the length of $\overline{BO}$ is 2/3 the length of $\overline{BM}$. This means that the altitude has length 4/(2/3) = 6. We now need to find the length of the base. Triangle BMC is a 30-60-90 right triangle. This means that the length of $\overline{CM}$ is $6/\sqrt{3} = 2\sqrt{3}$. The length of $\overline{AC}$ is twice this, or $4\sqrt{3}$. We can now find the area of triangle ABC:
$1/2 * 6 * 4\sqrt{3} = 12\sqrt{3}$.
The area of circle O can be easily found since the radius is known. $A = \pi(4^2) = 16\pi$. The area of the shaded region is the difference of these two areas. $16\pi - 12\sqrt{3} \approx 50.27 - 20.78 = 29.49$. Rounding this to the nearest tenth as requested gives our answer. **Answer: 29.5**

7. This problem can be solved by systematically labeling each vertex of the grid with the number of ways that it can be reached by the rat, working from point A to point B. The rat starts at A, so there is exactly one way to get there. Likewise, there is only one way for the rat to the points directly to the right of and below A. The vertex diagonally below A can be reached in two ways: by traveling right from the vertex below A or by traveling down from the vertex to the right of A. The diagram that we have created so far is shown below.



Because the rat can only move down and to the right, the number of ways the rat can reach a given vertex is the sum of the ways it can reach the vertices directly above and to the left of this. Using this rule, we can fill in the rest of the diagram.



Reading off of our diagram, we find that there are 126 ways for the rat to reach point B.
This answer can also be obtained by realizing that to go from A to B the rat must move along exactly nine segments, five of which are horizontal. There are $\binom{9}{5} = \frac{9!}{5!4!} = 126$ ways to choose which five of the nine segments will be horizontal. **Answer: 126**

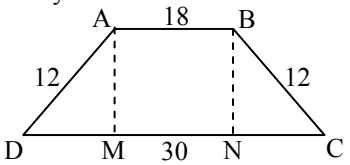
8. For Becca to have a chance at winning, Louis must guess wrong on his first term. There are 5 numbers for him to guess from, so there is a 4/5 chance of that happening. Since Becca is twice as likely as someone guessing to answer correctly, there is a 2 * 1/5 = 2/5 chance that she is correct on her first turn. So, Becca's probability of winning the game on her first turn is 4/5 * 2/5 = 8/25.
For Becca to win on her second turn, Louis must guess wrong on his first turn as before. Then Becca must answer incorrectly, which has a probability of 1 – 2/5 = 3/5. Louis must then guess wrong again. There are only four numbers to guess from, so the probability of this is 3/4. Finally, Becca must answer correctly. With four numbers the probability of this is 2 * 1/4 = 1/2. The combined probability of this occurring is 4/5 * 3/5 * 3/4 * 1/2 = 9/50.
For Becca to win on the third turn, as before Louis must guess wrong on his first two turns, and Becca must answer wrong on her first. Becca must also answer wrong on her second turn, with a probability of 1/2. Louis must guess wrong a third time, and with three numbers left this has a probability of 2/3. Becca must then answer correctly, doing so 2 * 1/3 = 2/3 of the time. So, the probability of this is 4/5 * 3/5 * 3/4 * 1/2 * 2/3 * 2/3 = 2/25.
If Becca wins on the fourth turn, everything must happen as before, except Becca must choose one of the two incorrect numbers, with a probability of 1/3. Louis must then guess incorrectly with two numbers remaining, this having a probability of 1/2. Finally, Becca must choose the correct one of the two remaining numbers, with a probability of 2 * 1/2 = 1. The overall probability of this happening is 4/5 * 3/5 * 3/4 * 1/2 * 2/3 * 1/3 * 1/2 * 1 = 1/50.
Since Becca is guaranteed to answer correctly on the fourth turn, the game cannot last any longer. The total probability that Becca wins is the sum of the four calculated probabilities:
8/25 + 9/50 + 2/25 + 1/50 = 3/5 = 60%.
Answer: 60%

Team Round

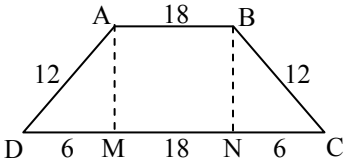
1. We are given that 1 m = 3.281 ft. Dividing both sides by 3.281 gives us that 1 ft ≈ .3048 m. Since there are 5280 feet in a mile, a mile is equal to 5280 ft * .3048 m/ft = 1609.3 m. **Answer: 1609.3 m**

2. Since there are two agents involved with the sale and each earns a 7% commission, the total commission equals 14% of the sale price. If this price is *x*, then we have $0.14x = 55440$ giving $x = 55440/0.14 = 396000$. We are next told that this sale price is 10% less than the assessed value. If we let the assessed value be *y*, then this means $x = 0.9y$ so $y = x/0.9$. Substituting for *x* gives $y = 396000/0.9 = 440000$. **Answer: $440,000**

3. We are given the lengths of the trapezoid's bases, so we only need to determine the height to find its area. We begin by drawing altitudes from A and B that meet segment $\overline{CD}$ at M and N respectively.



The two altitudes are parallel, so $\overline{MN} = \overline{AB} = 18$. ABCD is an equilateral trapezoid, so the triangles AMD and BNC formed at the two ends are congruent. This means that $\overline{DM} = \overline{CN}$. The combined length of these two segments is $30 - 18 = 12$, so each segment has length $12/2 = 6$.



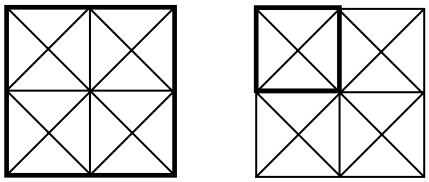
The altitude $\overline{AM}$ intersects the base at a right angle, so AMD is a right triangle. We can thus find the height, *h*, from the Pythagorean theorem. $h^2 + 6^2 = 12^2 \rightarrow h^2 = 144 - 36 = 108$ so $h = 6\sqrt{3}$. We can now use the formula for the area of a trapezoid:
$A = 6\sqrt{3} * (18 + 30)/2 = 144\sqrt{3}$.
Answer: 144√3

4. The number of employees has a remainder of three when divided by four, so it must be one

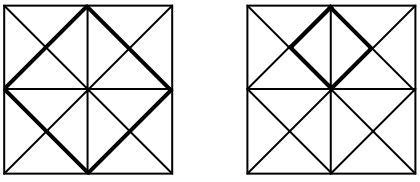
less than a multiple of four. Likewise, it must be one less than a multiple of both five and six. Therefore, we are looking for a number that is a multiple of 4, 5 and 6. The smallest number that satisfies this is called the least common multiple (LCM) of these three numbers. The LCM of 4 and 6 is 12. Since 12 and 5 share no factors, their LCM is 12 * 5 = 60. The number of employees is one less than a multiple of these numbers, so it is 59. This number is less than 100, so it is our answer. **Answer: 59 employees**

5. $10! = 10*9*8*\cdots*1$. We can determine its prime factorization by combining the prime factorizations of each individual term. These are given below:
$1 = \text{N/A}$ $2 = 2^1$ $3 = 3^1$ $4 = 2^2$ $5 = 5^1$
$6 = 2^1 3^1$ $7 = 7^1$ $8 = 2^3$ $9 = 3^2$
$10 = 2^1 5^1$.
The prime factorization for 10! is found by adding the exponents on corresponding prime factors:
$10! = 2^8 3^4 5^2 7^1$.
This gives $a*b*c*d = 8*4*2*1 = 64$.
Answer: 64

6. The entire outside of the figure is the first square, and this one is divided by the horizontal and vertical lines into four equal smaller squares.



There is another large square that is rotated 45° and inscribed in the large one. This second large square is likewise divided into four smaller squares by the diagonal lines.



There are no more squares contained in the drawing. The number of squares that we have listed is 1 + 4 + 1 + 4 = 10. **Answer: 10 squares**

7. We are not given the third term of the sequence (a_3) and there is no way to find it from the first two terms, so we will just let it be x. We can now use the provided formula to find the next three terms of the sequence:
$a_4 = a_1 - a_2 + a_3 = 4 - (-1) + x = 5 + x$
$a_5 = a_2 - a_3 + a_4 = -1 - x + (5 + x) = 4$
$a_6 = a_3 - a_4 + a_5 = x - (5 + x) + 4 = -1$.
It is interesting to note that this sequence consists of a repeating four number pattern 4, -1, x, 5 + x.
Answer: -1

8. The key to this problem is to realize that because the probe is continuously in motion, there is no need to keep track of its individual trips. Instead, all that is needed is the total time between when it is launched and when the ships meet, and with this information and the probe's speed its total flight distance can be determined.
The first ship flies at 50,000 km/h and the second flies at 80,000 km/h. They are flying directly towards each other, so they are closing the gap between them at a rate of 130,000 km/h. The ships are initially 400,000 km apart, so it will take 400,000 km/(130,000 km/h) ≈ 3.077 hours for the ships to meet. The probe has a speed of 120,000 km/h, so it will have traveled a total of 3.077 h * 120,000 km/h ≈ 369,000 km before the ships meet. **Answer: 369,000 km**

9. Tracy's clock is running fast, so the next time it will show the correct time is when it is a full 12 hours ahead of the correct time (12-hour clocks don't distinguish between a.m. and p.m.). There are 12 * 60 = 720 minutes in 12 hours, and because Tracy's clock moves five minutes further ahead of the correct time for every "hour" on her clock, it will take 720/5 = 144 "hours" before Tracy's clock shows the correct time again. However, each of her clock's "hours" is actually only 55 minutes long. This means that the clock will show the correct time again after $144*55 = 7920$ minutes or 132 hours of real time. **Answer: 132 hours**

10. The formula for the volume of a cylinder is $V_1 = \pi r^2 h$, while the volume of a cone is $V_2 = \pi r^2 h/3$. The problem says that the total height of the object is 2r, and each part of the object has the same height, so $h = 2r/2 = r$. Substituting this into the volume equations gives $V_1 = \pi r^3$ and $V_2 = \pi r^3/3$. The total volume is the sum of these volumes, or $V = 4\pi r^3/3$. This is equal to 288π so $r^3 = 288\pi*3/(4\pi) = 216$ and $r = 6$ inches. The diameter is twice this. **Answer: 12 inches**